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Learning of Geometry via Physical Activities in Snow

One of the main sources of learning difficulties in geometry among students from 12 years age and older, is probably the previous treatment of geometrical figures on paper during elementary school (Berthelot & Salin, 1998). In the teaching process, many students think of geometrical figures as if they were objects, while teachers refer to the same figures talking about geometrical concepts. In addition, Berthelot & Salin claim they have good reason to expect that geometrical knowledge is not spontaneously transferred to solve space problems.

According to Lev Vygotskij, *"To devise successful methods of instructing the school child in systematic knowledge, it is necessary to understand the development of scientific concepts in the child's mind."* (Vygotsky, 1999, p.146). The development of scientific concepts is closely connected to the development of spontaneous concepts: *"The child becomes conscious of his spontaneous concepts relatively late; the ability to define them in words, to operate with them at will, appears long after he has acquired the concepts. The child has the concept (i.e., knows the object to which the concept refers), but is not conscious of his own act of thought."* (ibid. p. 192).

Background

Old Norwegian proverb says that Norwegians are born with their skis on. What happens to a child learning how to ski? One of the first experiences of the child is that skis need to be parallel to fit into the trail. It is quite difficult for the child to keep skis in a parallel position; in the beginning the skis are crossing each other over and over again resulting that the child will stumble and fall. When the child is able to make these parallel moves in the trail, the next step is to perform some trails on her/his own. The proud child then is turning her or his head backwards admiring the pair of self made parallel lines.

The children's experiences are body experiences, not verbal experiences. Most of these experiences become automated moves; one of the body's ways of releasing brain from unnecessary work. Small children normally do not speak about their experiences in terms of geometrical concepts. They have a fundament for spontaneous concepts about parallels; their skis have to be parallel to fit into the skiing trail. They have a fundament for spontaneous concepts about angles, too; the smaller the angle is between the skis, the easier the skis fit into the trail.

From a Euclidian point of view, two lines are parallel if and only if they do not intersect each other. School geometry often introduces the parallel concept by showing pupils visual examples of parallelism or by presenting them for two parallel lines drawn on the blackboard. The students' perception of parallelism then is reduced to a passive visual glance at figures with the parallel property. Afterwards pupils draw parallel lines themselves. School geometry does not seem to have in mind the pupils' total perceptuated experience.

Questions

Could it be that the children's development of scientific concepts in geometry would increase if their experiences from performing figures and patterns in snow were connected to their performing figures and patterns on paper? Our Norwegian mathematics teaching tradition is not known for using children's total knowledge and experiences as a fundament. Berthelot & Salin (1998) supports our present curriculum, however, by pointing out that children's own experiences are essential in mathematics teaching. Could it be that the pupils' expectations to their own learning ability in mathematics would increase, if the teaching were based on outdoor activities familiar to the pupils? This kind of teaching agrees with the curriculum. But what about teachers? Some of them want to learn how to let their mathematics teaching follow the curriculum, but most teachers do their teaching in ways they are convinced is the best. Probably they will not change their way of teaching just because some researcher say it could be a smart thing to do.

One possible way of changing mathematics teaching tradition, is to give teacher students and teachers some good experiences; to make them able to discover some of their own hidden geometrical knowledge. Knowledge related to their experience from physical activities. Could it be that teacher students increase their understanding of didactics by performing physical activities in snow during mathematics lessons?

Theory

To most public mathematics is largely invisible and unrecognized (Niss, 1994). Even some mathematicians and mathematics educators do not have a clear picture of the role of mathematics in society. The key to explaining this could lie in the fact that mathematics is not found on the surface of the matters to which attention is paid. "*The mathematics is invisible because it is hidden, not because it is absent*" (ibid. p 372). If teacher students experience they are able to discover some hidden mathematics, then maybe they think there could be some more mathematics for them to discover. It is necessary for teachers to see the mathematics invisible to them before they are able to include this mathematics in their teaching.

I suppose we need getting conscious about the size and dimensions of geometry in our teaching:

...natural knowledge of space is strongly structured into three main representations: *microspace* (corresponding to the usual prehension relations), *mesospace* (corresponding to the usual domestic spatial interactions) and *macrospace* (corresponding to unknown city, maritime or rural spaces...) In consequence, the space representation produced by the usual out-of-school experiences is not naturally homogenous, and is quite different from elementary geometry. (Berthelot and Salin, 1998, p 72)

Schools play an essential role in children's development of mathematical knowledge. However, mathematical knowledge develops not only through school instruction but also as a result of participation in out-of-school activities (Schliemann et al, 1998).

According to van Hiele's levels of geometrical thinking (Fuys et al, 1988) students at the lowermost level identifies and operates on geometric figures according to their appearance. At the next level the student discovers properties of a class of shapes. The small child able to go skiing already knows quite a lot about properties of parallelism. Children often have got this experience before they are able to produce an single sentence of words on their own.

Language plays an important role in the development of geometrical thinking. "*In stressing the importance of language, van Hiele notes that many failures in teaching geometry result from a language barrier – the teacher using the language of a higher level than is understood by the student.*" (ibid. p. 7)

Performing physical activities in snow during mathematics lessons gives teacher and students possibilities of non-verbal communication. The non-verbal communication can support the verbal words and the verbal words can support the non-verbal communication, too. Combining verbal and non-verbal communication can make it easier to the teacher to avoid using a language at a too high level for the students.

According to the conceptual framework of the OECD/PISA assessments: *“Mathematics is the language that describes patterns, both patterns in nature and patterns invented by the human mind. In order to be mathematically literate, students must recognise these patterns and see their variety, regularity and interconnections.”* (OECD, 1999, p 48). Children, who are familiar with skiing and playing in snow, also are familiar with patterns as fishbone and other common ski-trail patterns.

Case I – special education in elementary school

Two young boys at 8th and 9th grade had their mathematics lessons in a classroom on their own, alone with their teacher. They did not take part in the ordinary class' lessons. These boys did understand quite a lot of mathematics. For example, both of them could see and express the connections in Euler's thesis after two hours of work with polyhedrons. They had never worked with expressing mathematics with letters, but they were able to express themselves with some words and gestures. If the teacher asked questions like "How much is one half of 176?" Both of them were able to answer correctly within a few seconds. Their problems were to understand the meaning of concepts. Both of the boys were diagnosed to autistic disorder.

It was difficult to these pupils to understand what a meter was. Both of the boys did a lot of systematic traditional work on the meter. One of the boys had his height marked at the wall in his classroom. In the middle of winter he was asked by the mathematics teacher, "Do you know how tall you are?" The boy was thinking for a while – then he answered "twenty-four zero nine." The teacher then asked him to explain this some further. The boy replied that it was written like that on the wall – the boy's height was measured the 24th of September. The boy remembered the numbers quite well, but he did not understand anything about why these numbers were written.

Skiing was a regular activity in these boys' lessons during wintertime. One of the boys was in need of special physical training; among other things he was an ambler. To exercise in diagonal walking, he was skiing by following the teacher's trail. Some mathematics was included in the skiing lessons as well.

After about two or three lessons the boys were able to talk about parallels. They could tell that the trail was two parallel marks made by their skis; the electricity cables in the air were parallel and so on. After a while the teacher walked beside the pupils, making two more parallel lines. The pupils were asked if these marks were parallel, too. The boys agreed spontaneously that now there were four parallel marks and that the distances between the two lines in the middle was greater than the distance between the other lines.

In the classroom the boys were able to write down sentences about parallelism, they could draw parallel lines on a paper and they could draw even three parallel lines on one paper. Their drawn parallels were observed not to be parallel to the paper's edges. The boys also could tell about parallel lines elsewhere in their surroundings. It seemed as if the boys were able to make use of three dimensional experience from meso space in more generalised contexts on paper in micro space. This seemed to support the idea that the boys' scientific concept of parallelism was increased.

Then the *angle* was introduced, as the position of the feet while a person is going skiing uphill, fishbone going. A steep slope needs a large angle between the skis; a less steep slope needs a smaller angle between the skis. Elsewhere, you slide backwards. Understanding

what means with an angle is often difficult to pupils; there are a lot of possibilities of misunderstanding.

Skiers have a special technique used for turning backwards in her/his own trail. The skis are lifted and turned one by one, and the skis have to be lifted quite high not to make any disturbing marks in the snow. Both of the boys exercised in performing a 180 degrees' turn, and both of them managed to perform this turn. The boys were told that the name of this move was "to turn 180 degrees". If they managed to make an almost complete turn, then they suggested the turn was a little less than 180 degrees. In this situation we were using the word *turn*, not *angle*. According to Brekke et al (1998) focus on 180° as half a turn could help pupils getting a better angle concept. It has been problematic to Norwegian pupils to think of 180° as an angle.

These two boys did not seem to have any special trouble in understanding the outdoor work with neither angle nor parallel. This could be due to the way of teaching, but it could be due to many other reasons as well. For instance their understanding could be due to that these concepts were new to the boys when they were introduced in these particular lessons. As teachers, however, we could see that this teaching worked out fine with these two pupils.

Case II – University college teaching

The skiing mathematics lessons in special education seemed to be quite successful and led to the idea of introducing teacher students to the same kind of lessons. After one pilot year, all stationary teacher students at our university college were offered one skiing and mathematics day, led by assistant professors from mathematics and from sports and movements. As a follow-up from the outdoor activities students were given a written task: "Sketch a teaching program that focuses on teaching geometrical concepts in elementary school. Make use of snow activities as a basis for the teaching. The teaching program has to be meant for one specific chosen class." A few weeks earlier these students had been teaching mathematics at elementary school and most of them chose to sketch a teaching program meant for pupils at the same age.

42 out of totally 97 teacher students, 43% of the total group, participated in the outdoor day including skiing and geometry. This program was at their ordinary schedule, and as for the rest of the mathematics lessons, participation was voluntary. The weather was not especially nice, it was windy and some snow was falling, too. Students need to prepare to take part in such lessons; they needed enough and useful clothes, and they needed to bring skis and sticks with them, too. To go to a lecture, students do not need any preparation at all.

The following week 59 students participated in the mathematics lecture. At the end of this lecture the students were asked to range the utility of the outdoor day in a given scheme. 34 of these 59 students wrote they had positive experiences from this day and 19 did not participate in that day's work. One of the students, who did not show up the outdoor day, wrote: "I did not participate because of an injury". It seemed quite clear that the students, who participated, meant this was an utilisable day.

At the annual evaluations at our university college, lectures always get a high score when students are asked to give their opinion about different kinds of teaching. How useful did this group of students think the ski-and-mathematics day was compared to the lectures? Table 1 shows their opinion of this.

Outdoor in snow	Outdoor and lecture equal advantage	Lecture	Did not participate in the outdoor day	Total amount of students responding
18	15	7	19	59

Table 1. What kind of teaching students believed led to most advantage.

This figure tells us that among these 59 students 33 mean they have equal or better utility from ski-mathematics day than from ordinary lectures. This is more than half of the total group – the other ones, including those who did not participate from different reasons, counted 26 totally. More precisely, only 7 students were sure they had more advantage from listening to a lecture than from taking part in the out-door program.

Six months later the same group of students were asked some questions about their belief in this kind of teaching. The students were questioned in the end of a random lecture in some other topic. This time 39 students were present. 22 of these participated in last year's outdoor day and 17 did not. Their answers showed that 13 of the participants believed this outdoor day made the topic of mathematics more interesting and 14 of them believed it increased their understanding of didactics. The students were not so sure they could make use of their experiences from this day; many of them felt they needed more mathematical knowledge to do so.

As mentioned, 42 students participated in the outdoor activities. 30 of these students wrote didactical texts afterwards. One of the questions six months later was: "If you wrote a didactical text in connection to the outdoor activity day in snow, did you think working on this text increased your didactical understanding?" Table 2 shows the answers, only 2 of them were not sure if they had learned some didactics from working on this text. Almost all the writers had learned some didactics from this work.

Yes	Maybe	No	Did not write text	Total number of students answering the questions
18	2	0	19	39

Table 2: Students' opinion about if they learned something from writing the text in didactics.

Those students who did participate in neither the text writing nor the outdoor activities were given some questions, too. One of these questions was: "Do you think those who participated learned some didactics this particular day?" All of the answers were "Yes". The challenge for teacher educators lies, however, in the following: The students who did not participate in the outdoor teaching program, were given one more question: "Do you think you would participate this year if you were offered a similar outdoor day?" Twelve of these nineteen respondents were not sure if they then would participate, even though they believed participating would result in learning of didactics. A common student declaration is that they need more didactic teaching, but it seems to be difficult to educators to make all students motivated.

Case III –teacher students' teaching

The following year almost all teacher students participated in one outdoor day in late autumn. The compass was essential in this program due to the idea that compass is a useful tool for working with angles as dynamic topics. Using the compass is an activity that is worked out more easy if the users co-operate with others than if they work individually. Teacher students wrote down their teaching ideas as a didactic text before they had done any teaching on their own. The idea was to give teacher students some time to reflect upon how to teach mathematics as early as possible during their teacher study.

One of these groups chose a teaching program for their period of practice that included outdoor and indoor geometry in 7th grade. Their teaching program included use of compass,

too. According to these aspects, after half of their practice period the group was given a closer follow up including response to their teaching, letters and interviews.

The group of students counted five persons with a variety of attitudes towards mathematics. In the beginning, one of them did not even believe in this kind of teaching at all, and she was not fond of outdoor activities, either. She made up her mind to give this project a try; after all she was part of a group. When students told teachers at their practice school about their mathematics teaching-program, they were met with scepticism and laughter.

Only two of the students were familiar with the compass before they got started, the three others did not know anything about how to use a compass. Thus they asked the experienced ones to be responsible for the part of the teaching concerning the compass. But then, what if pupils asked some of the others about how to use the compass? Students made up their mind to try teaching each other about the compass; they could not introduce to pupils something that was too difficult to teacher students to understand. The complete group of students then spent some hours in the gym working with the compass and helping each other to understand how to use it. To be able to take an active part in the teaching, all five students needed to see the mathematics “hidden” in the compass before they could be able to include this mathematics in their teaching.

The teaching program started with introduction of compass – how it worked and how to use it. The children were asked what mathematics they could find in a compass. According to students, the pupils’ answers were circle, angles, degrees, radius, diameter and rotation. By giving pupils such a question, students gave pupils opportunity to start analysing the mathematics content of the forthcoming activity. Several of the pupils’ scientific concepts were challenged, too. The students were aware of searching for eventually hidden mathematics, and they let the children use their own words about the compass’ mathematics. It is not obvious to children that use of a compass is use of mathematics. That is not obvious to all mathematics teachers either.

Then the complete group continued working outside school. All outdoor activities took part in the three dimensional mesospace (Berthelot & Salin, 1998). Pupils were placed in two rods. The student who performed this lesson asked pupils to turn 90° to the right, then 180° to the left and so on. By performing these turns, students focused both on pupils’ spontaneous and scientific concepts about turning. The students argued they did it this way to see if pupils had understood the explanation given in the classroom. I think these actions gave all pupils possibility to start understanding what to do and how to do it. It was natural and not forbidden to pupils to watch each other and to copy other students’ moves. The teacher student asked the pupils to turn and turn, to the left and to the right, 270° , 180° and so on, until the pupils acted as a synchronised group. By doing this, after a while the students could see that all the pupils then understood how to perform these turns. At that moment the entire class was able to carry out some mathematics exercises in a correct way. All pupils then had some knowledge about turning and degrees in one specific context. They had some common mathematics experience that could be used as a fundament for classroom teaching.

Next, pupils were given different written geometrical tasks placed in the environment. To find the tasks, each group of pupils had to use the compass to find and follow the correct direction to it. Pupils started from a given point and had to go back there when they should find direction to the next task. One of the students was standing at this given point to guide pupils in using compass when guiding seemed to be needed.

In their final report the students write: “*After having observed pupils for these two lessons, we were confirmed that pupils had learned something from the outdoor day. It was exiting to see how outdoor activities were used in connection with mathematical activities in the classroom.*” (Solhaug, Skau, Sørboe, Valanes og Kind, 2003, p 9, my translation). The student who did not believe in the project wrote afterwards: “*As I see it, this was a success*

among pupils. It is my opinion that all of them had learned something from this teaching program. It was real nice to watch pupils' engagement and to which extent they used outdoor experience while working with mathematics afterwards. I guarantee I will make use of this kind of teaching when I become a teacher."

Students write in their report that pupils' attitudes towards mathematics seemed to be better from the outdoor working; the pupils were really engaged in the tasks and they enjoyed the work. Afterwards, in the classroom, students were given lots of possibilities to talk and discuss geometry with the pupils. During classroom work afterwards all of the students observed pupils making use of outdoor experiences while working with mathematics. All students agreed that it was necessary for them as mathematics teachers to take an active part in the outdoor activities. Every one of the students told they had given hints to pupils when pupils did not at first glance see the connections between outdoor and indoor exercises. Students did not tell pupils about these connections, by giving hints they led pupils to find the connections themselves.

Concluding question

Is it so that performing outdoor physical snow activities in the mathematics lessons can give rise to better geometrical understanding? The answer can be yes, but there are several difficulties both in how to perform the out-door activities and in how teachers connect the following classroom work to pupils' experiences from out-door activities. Some teacher students – and teachers as well – do not take part in out-door activities. Thus there is reason to believe they do not engage in this kind of teaching.

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