UiT Aurora Center for Mathematical Structures in Computations (MASCOT)

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Summary

The Aurora Center MASCOT will unify and advance research frontiers in the interplay of pure mathematics and computations by exploring fundamental mathematical structures and their increasing significance for modern algorithms and novel program architectures. Indeed, abstractions and ideas from pure mathematics generate crucial insights into the structure of computational algorithms and discretization techniques underlying numerical modelling and physical simulations. But large computations have also become more relevant for research in pure mathematics recently, as increasing computing power offers new possibilities to explore abstract structures and rigorously prove theoretical results that have been out of reach in the past due to their complexity. Between these two poles of interactions the proposed Aurora Center will unite experts to reveal and exploit fundamental structures of mathematical computations, explore the algebraic and geometric nature of computational complexity, and analyze structural methods to reduce this complexity, as well as adapt these methods to concrete applications in optimization, control theory and robotics.

Three research directions will be the guiding principles within six cross-disciplinary work packages, focusing on the interplay of mathematical structures and computations: (1) Optimization of computational processes using symmetry methods and structural analysis, (2) Exploration of efficient computations to complex problems in algebraic combinatorics, geometric integration and nonlinear systems, and (3) Applications of mathematically structured computations to control theory, geometric robotics, automation, material science and modelling. Each pair of these directions has seen some cooperation before; our far-reaching goal is to unite them into one possible triangle (ref to the Fig 1: we aim to create new shapes through nonlocal cooperation and synergy).

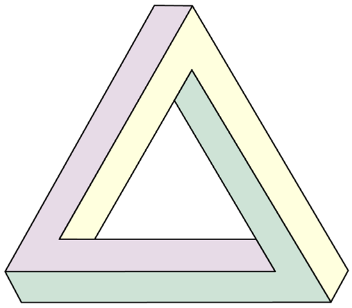

Fig.1: Impossible triangle by R Penrose[1]

Both senior and junior scholars are represented among the PIs. This will ensure future recruitment and fulfil the intention of the Aurora call, namely preparing the team members for further grant proposals, such as a Center of Excellence (CoE) and European Research Council grants (ERCs). Our overall ambition is to establish a scientific Research Center of top quality, which will be unique nationally and renowned internationally for its role as a bridge between pure mathematics and applied computations.

1. Excellence

1.1. State of the art, knowledge needs and project objectives

Mathematics is an inherent part of the everlasting human endeavor to understand the world beyond its obvious physical presence and limitations. Computation can be seen as one of the pillars of modern science, with its success stretching from pure and applied mathematics all the way to the quantitative sciences. In the last two decades, the interaction between pure and computational mathematics developed an enormous potential for advancing fundamental, long-standing questions in the interplay between mathematics and computer science.

The works of the recent Abel Prize winners[2] László Lovász and Avi Wigderson (to be awarded by the king of Norway Harald V in May 2021) are examples for this successful connection of mathematics and computations.

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[1] Sir Roger Penrose is a mathematician who received the Nobel Prize in Physics in 2020. Investigation of his CCC scenario for the universe compactification is a WP in the GRIEG project SCREAM, where BK and DT are PM/PI: see cft.edu.pl/grieg.

[2] "for their foundational contributions to theoretical computer science and discrete mathematics, and their leading role in shaping them into central fields of modern mathematics" (citation: the Abel Committee, chaired by H. Munthe-Kaas).

While computational challenges are manifestly discrete, we will also be interested in continuous models that allow a convenient discretization (quantization) and which in turn can be obtained in a (quasi-classical) limit. Currently this is known as geometric complexity theory. It is an open question which discretizations are effective for such models, whether dynamical properties or integrability depend on the discrete-continuous paradigm, and which are better reflected in real-world applications.

The goal of the Center is the unification of pure and computational mathematics to arrive at the creation of new connections and principles, which allow new computational approaches to fundamental questions arising within mathematics as well as from practical applications. The research will be focused on optimization of computations using symmetry methods and structural analysis, with applications to algebraic combinatorics, geometric integration, nonlinear systems and control theory, robotics and modelling. The Center will serve as the nucleus for a leading national Center of excellency that will be established in collaboration with national partners and extensive contacts worldwide.

Three research directions (RD) will be in the focus of development at the Center.

RD1: MFC (Mathematics facilitates computations). Many problems stemming from applications present some built-in symmetry and hence appeal to applications of group representation methods in computational mathematics, ranging from numerical linear algebra to Fourier analysis on compact and unimodular Lie groups. This has not been pursued to large extent within theoretical computations. Additionally, the understanding of Hopf algebraic structures and their importance in numerical analysis, and stochastic differential equations has increased (see [1]): PreLie and PostLie algebras, the Butcher-Connes-Kreimer (BCK) and Munthe-Kaas Wright (MKW) Hopf algebras relate combinatorial structures for producing high quality software, as is developed in a current NFR project by Munthe-Kaas. One overarching objective of the MASCOT Center will be to evaluate and reduce computational complexity via geometric and algebraic structures.

RD2: CCM (Computational challenges in Mathematics). The recent decade saw several breakthroughs on long standing open problems thanks to a revolution in computational capacities. The celebrated proof of Kepler's conjecture by Hales [2] can be seen as an example. Yet brute force has a limited range and an intelligent design of computations using advanced symbolic systems is required. One example where this approach was successful is the recent solution of the symmetry gap problem for geometries, based on large groups such as exceptional Lie group[3] E8, by Kruglikov and The [3]. A focus of the MASCOT Center will be to attack theoretical problems, ranging from supersymmetric models, algebraic statistics, and space homogenization to rigorous justification of dispersionless integrability and new algorithmic tests via high complexity computations.

RD3: RWA (Real world applications). The mathematical methods and computational techniques explored within the Center will also be applied in a variety of practical problems. Among the most important for the scope of our project we will consider: automation and robotics, smart energy and power systems, porous media and composite material design, cybersecurity and information transmission problems. The methods applied in these tasks are versatile, and within the framework of the MASCOT group we aim to obtain a common working ground, where the new ideas for applications are derived and new theoretical methodologies are developed. The interdisciplinary research and collaboration of the various nodes in the Aurora Center should result in synergy of computational approaches and advance in applications.

1.2. Novelty and ambition

Recently, there has been an essential progress in understanding the mathematical mechanism of symmetry breaking, with many highly symmetric homogeneous geometries classified. The computational tools behind this involve representation theoretic and homological techniques. High degree of symmetry also helps in simplifying optimization problems and unveiling the algebraic structures in statistics. Surprisingly, some of these Lie algebraic structures have been also discovered in B-series expansion methods of computational mathematics. This has been applied to solutions of differential equations, but undoubtedly these structures will prove useful also in more general integral-functional equations.

The research to be carried out at the MASCOT Center will not only explore these novel ideas but will also find applications to the fundamental mathematical methods in some important real-world problems. A combination of these tools is the innovation that our team is capable to accomplish in the proposed timeframe.

[3] The proposed Aurora Center Logo shows a two-dimensional projection of the eight-dimensional root system of E8.

Timeliness relative to recent advancements in both pure and computational mathematics in conjunction with focusing on real-world problems will ensure an impact extending far beyond the Center itself. Our ambition is to take a leading role within the mathematical community by substantially strengthening and expanding the bridge between mathematics and computations and to make UiT an important node in these developments through understanding and utilization of fundamental structures in mathematics.

1.3. Research questions and hypotheses, theoretical approach and methodology

Research problems are partitioned into 6 work packages in the project as follows. In theoretical parts the hypotheses are exceeding but we indicate the methodology, while for practical applications we briefly name the hypotheses and the theoretical approach. For brevity's sake, we indicate only central PIs involved.

WP1: Combinatorial Algorithms & Data Security (PIs: Johnsen, Mohammadi, Riener)
This package is an interplay between RD1 and RD2 with the following objectives:

Obj1.1: Structural properties of the control-flow graph for polynomial systems.
In recent years, techniques and concepts of modern algebraic geometry have shown to be relevant for problems in applied mathematics and computer science to solve real-world problems. Several state-of-the-art algorithms in artificial intelligence and correctness verification of computer programs rely heavily on solving systems of polynomial (in)equalities or finding invariants of their respective varieties. Currently, most algorithms assume that the resulting systems are in the most general form, which makes them inefficient. The main goal of this objective is to derive efficient algorithms for special systems arising in applications to computer science by using underlying combinatorial structures. Taken a system of polynomial (in)equalities we associate to it a graph as follows: the vertices correspond to the variables and an edge between two vertices signifies that the corresponding variables appear in the same polynomial.

A challenge is to compute algebraic invariants of the variety defined by the system, for example its degree, which can provide important insights into the complexity problem. In applications, the system is constructed via the control-flow graph (CFG) of the program and the associated graph inherits structural properties of the CFG [4]. Often, such graphs have low tree-width [5,6]. Thus, it is apparent that methods to exploit combinatorial properties of the underlying graph structures are highly relevant. Our goal is to obtain such algorithms by studying connections between the combinatorics of the graph and properties of a Gröbner basis of the system. This naturally connects previous lines of research by Mohammadi and Riener. We will apply our knowledge to solve concrete semi-algebraic type systems.

Obj1.2: Error correcting codes and information transmission problems.
The classical theory of error-correcting codes is the result of joint works by computer scientists, information theorists and mathematicians. The main purpose is to find efficient ways of sending digital information through a noisy channel, such that the receiver can discover errors due to the noise and retrieve the correct information, provided the amount of disturbance is limited. This theory has a great relevance wherever digital information is sent, including a wide variety of applications, like satellite communication, TV transmissions, and designs of registration numbers. In this objective we aim to explore recently established connections of coding theory to the theory of matroids and polymatroids, arising from the works of Johnsen and collaborators in a recent cooperation project with IIT-Bombay [7,8].

The theory of matroids provides a combinatorial framework, which has already shown to be highly relevant in many parts of mathematics and theoretical computer science. The main observation behind the work of Johnsen was that one can represent each generator and parity check matrix of a linear code by a pair of representable, dual matroids. These matroids allow to understand and study certain parameters of the code, which are relevant for the transmission capacity of a code, for example, the so-called Hamming weight. This line of work has proved to be extremely fruitful, resulting in a better understanding of coding theory. More generally, we also aim to explore different combinatorial structures inherent in certain codes and their relation to algorithmic tasks like decoding and encoding. One example of such structures which will be explored are q-matroids and rank metric codes, which provide new analogues of classical error-correcting Hamming codes.

WP2: Symmetry, Invariance and Image Recognition (PIs: The, Kruglikov, Munthe-Kaas)
This package is an interplay between RD2 and RD3 with the following objectives:

Obj2.1: From classical symmetry to computational supersymmetry.
A broad research interest among the Geometry group at IMS concerns symmetries of geometric structures. There have been numerous recent theoretical advances on the pure mathematics side developed by Kruglikov, The and collaborators on symmetry dimensions and gap phenomena (see [3] and follow-up refs), Cartan classification methods, and extensions of these to the supersymmetric setting. In the classical setting, this was significantly facilitated through excellent computational software functionality for differential geometry (Maple) and representation theory (LiE). These tools allowed for substantial experimentation, efficient testing of conjectures, and excellent training frameworks for our students and fellow researchers.

In contrast, computational support in supergeometry is currently severely limited. We propose to remedy this situation by focusing on the development of computational tools analogous to those in the classical setting. This includes: (1) Extension of Maple's JetCalculus package to allow for odd (fermionic) variables for the efficient computation of PDE supersymmetries and geometric realizations of simple Lie superalgebras; (2) Extension of Maple's LieAlgebras package to support super-functionality, allowing to compute cohomology, the Tanaka-Weisfeiler prolongation, and fundamental curvatures controlling super-deformations of a given supergeometric structure; (3) Investigation of supersymmetry gap phenomena and classification of submaximally symmetric models via Cartan-theoretic computational techniques.

Obj2.2: Complementary approaches to shape analysis and image recognition.
Both classical and differential invariants of Lie group actions have found applications in image analysis. Despite sensitivity to perturbations, they can be regularized to provide stable integral invariants. The signature manifold is a convenient tool to compare different objects and structures. Usage of group theoretic methods to compute invariants has become a trend and the team includes experts in this field [9]. Discretization of those methods relate to the modern development of machine learning. We will combine Lie theoretic methods with the ideas of stochastic geometry to enhance the approaches to problem of image recognition.

On the other hand, conformal geometry provides powerful analytic tools to deal with shapes via Teichmüller theory and uniformization technique. This again has a counterpart in differential invariant theory but now of infinite-dimensional groups modulo maximal Lie subgroups. It remains an open challenge to explore the relation to integrable systems (namely Gibbons-Tsarev systems) via stochastic Löwner evolution[4]. Discrete conformal invariants played a significant role in the solution to critical percolation problem by the Fields medalist S.Smirnov. We aim at applying discrete Löwner evolution and discrete integrable systems to shape analysis and computations.

WP3: Real Algebraic Methods in Optimization (PIs: Riener, Mohammadi, Bordin)
This package is an interplay between RD2 and RD3 with the following objectives:

Obj3.1: Structural approach to real algebraic optimization.
Real algebraic geometry deals with sets defined by a finite system of polynomial equalities and inequalities. Such sets are called semi-algebraic and since they are ubiquitous in applications, the study of their quantitative properties is increasing every day. The algorithmic study of semi-algebraic sets provides solutions to various problems arising in optimization, robotics, computer vision, automated theorem proving and more. There is a natural connection between semi-algebraic geometry and polynomial optimization: to maximize/minimize a polynomial function over a set given by polynomial inequalities. Using basic theorems from real algebraic geometry, one can approach such NP-hard optimization problems via semidefinite programming (SDP), a powerful generalization of linear programming to matrix variables.

The fact that SDPs can be efficiently solved by interior point methods makes SDP an interesting optimization paradigm. However, the bottleneck is the size of the resulting SDPs. In view of the present status of numerical solvers, the applicability of the basic methodology is limited to small or medium size problems unless some specific characteristics are considered. Riener, together with collaborators, has developed various approaches to exploit algebraic structures in these problems [10]. We aim to further refine these techniques and apply them to fundamental problems in geometric optimization, namely computing the chromatic number of discrete spaces, obtaining bounds on the sizes of codes and point configurations optimal with respect to a given property. These methods will be also used in the frame of cooperation on the second objective (below).

[4] As a surprising application, Lawler, Schramm and Werner used SLE to prove the Mandelbrot conjecture claiming that the boundary of planar Brownian motion has fractal dimension 4/3.

Obj3.2: Smart energy, power systems and optimization modelling.

Energy Informatics is an emerging interdisciplinary science that lies at the intersection of such areas as power systems, economics and computer engineering. It studies information and communication technology means to effectively manage fossil and renewable resources. Modelling of energy and power systems is a key area of Energy Informatics [11]. The main reason to investigate large-scale energy and power systems optimization models is the high attention that is currently given to the energy transition towards a low carbon future for the energy sector. This was a recent focus of research of Bordin [12]. There is a need to move from the so-called technology-agnostic to technology-oriented models, but the inclusion of technological peculiarities brings along an increased computational complexity (due to multi-dimensional variables, non-linear formulations, etc.) that can penalize the use of power systems models. This emphasizes the need to investigate possibilities to improve the representation of certain energy-related technologies within mathematical optimization. The common challenge is that the applications to large-scale instances require high computational time to reach a solution. Computational complexity of such large-scale energy and power systems models remains an important challenge which will be resolved with methods developed in the Center.

WP4: Nonlinear Systems and Mathematical Physics (PIs: Kruglikov, Piatnitski, Erlandsson)

This package is an interplay between RD1 and RD2 with the following objectives:

Obj4.1: Dispersionless integrable systems with applications to hydrodynamics and mathematical relativity.

Solitonic-like equations are the base for our understanding of infinite-dimensional integrable systems, fundamental in many applications. Their dispersionless limits inherit the integrability, but this has a special geometric flavor not visible in full dispersive context. The correspondence between different integrability approaches is a topic of active research interest. This connects such distinct fields as complex geometry, hydrodynamic reductions, gauge theory and general relativity. During the latest years Kruglikov with collaborators established a variety of new results unveiling remarkable relations between formal theory of differential equations, twistor theory, parabolic and cone geometries, self-dual gravity and conformal backgrounds. This constitutes the core of the research project Integrability via Geometry[5]. Calculating the integrability criteria is a computational challenge that requires a considerable computer power and time. The up-to-date progress encompasses the most important physical dimensions three and four. In alternative approaches, like the string theory, one studies higher dimensional systems, but this is demanding both from a purely mathematical viewpoint and from the computational side. We aim to progress in this direction by focusing on dispersionless systems with four non-holonomic dimensions and higher real codimension. There are examples, supporting the conjectural extension. A related analytic problem concerns the spectral properties of the associated integrable hierarchies.

Obj4.2: Evolution of global/nonlinear systems and the problem of complexity.

Dynamics of geodesic flows is the central theme in theoretical and celestial mechanics. Invariants like entropy help to distinguish between chaos and integrability paradigms for algebraic systems, though in the general smooth case the relation is more complicated. Polynomial integrability is a strong constraint on Hamiltonian systems even locally. A computer-based approach to this problem was developed by Kruglikov and Matveev, and successfully applied to the investigation of some potential candidates for black hole cosmology [13].

Globally, the behavior of geodesics is much more complicated. A particularly important question of existence and number of periodic orbits of specified type was a subject of recent study by Erlandsson [14], building on the preceding work by M.Mirzakhani[6]. Here two periodic orbits are of the same type if they differ by a mapping class. For surfaces of genus g, the number of such orbits of complexity $\leq L$ (e.g., length of the geodesic) is asymptotic to a constant times L^{6g-6} (this bound is related to the entropy h of the Teichmüller flow on moduli space [15]). Moreover, the periodic orbits can be shown to be equi-distributed in the Bowen sense. We aim to replace the mapping class group by any subgroup G and search for an analog of entropy h to bound the complexity growth. The key point is to construct a measure, mimicking that of Patterson-Sullivan in the setting of Kleinian groups, however a technical difficulty to overcome is the non-linearity of the group action.

WP5: Geometric Robotics, Analysis and Automation (PIs: Solvang, Piatnitski, Bordin)

This package deals with applications RD3 and is sub-divided into the following objectives:

[5] https://www.researchgate.net/project/Integrability-via-geometry

[6] The first woman who won the Fields medal (the highest distinction in mathematics) as well as the first Iranian woman whose photo without a traditional hijab was published in the official newspapers despite the country's strict rules.

Obj5.1: Mathematic optimization for real-time automation and logistics operations.

The manufacturing sector is considered as the backbone of economic development. The future generation of advanced manufacturing strongly depends on intelligent automation and logistics solution, which can benefit from the interaction between the physical and its cyber systems (Cyber-Physical system, in short: CP). In other words, a manufacturing system should always have a digital twin (DW), see [16]. This DW model will be effectively used for the purposes of planning, monitoring and providing real-time optimized solution based on highly complexed multicriterial and stochastic information and data collection during operations. The work package will focus on the mathematical solution base for developing multicriterial, non-linear and stochastic models for real-time optimization of a manufacturing system. In addition, based on the results and experience by Solvang, the developed mathematic models will be connected to their physical system for information sharing and data collection through cloud-based and middle-ware regulated platform [17]. Industrial Internet of Things (IIoT) will be applied for closing the CP system loop while suitable Application Programming Interface (API) will be developed and provided as implementation interface for end-users.

Obj5.2: Composite materials, homogenization and porous media.

Many modern branches of engineering and applied sciences face the necessity of macroscopic description of materials with a microstructure. This may be represented as fast oscillation of the characteristics of the environment or have rapidly varying geometry. Piatnitski has studied such problems, which appear, for example, in porous media and composite material design. Typically, in the artificial materials the microstructure is periodic, while in the natural materials it is random stationary [18]. The equations that describe such environments have rapidly oscillating coefficients in domains with a complicated microscopic geometry. In many cases, direct computational methods either fail to work for such problems or lead to expensive computations. Homogenization theory is a mathematical discipline whose goal is to derive the equations directly for the macroscopic (effective) characteristics of the environment. Nowadays it is a well-developed theory that comprises many methods, approaches, and rigorous results. Recently there has been growing interest in various ecological and population dynamics models where the environment is described in terms of non-local operators of convolution type. The objective is to develop the homogenization theory for such non-differential convolution type operators.

WP6: Complexity, Computations, Structure Preservations (PIs: Munthe-Kaas, Mohammadi, Riener)

This package deals with challenges in RD1 and is sub-divided into the following objectives:

Obj6.1: Mathematical structures in computations.

Algorithms for simulating dynamical systems is a central topic of computational mathematics, with countless applications in robotics, physics and engineering. Recent developments have focused on structure preserving geometric integration algorithms [19]. These are discretization techniques that exactly preserve geometric features, symmetries, and conservation laws of the dynamical system. In particular, the research activity of Munthe-Kaas has pioneered the use of Lie group techniques in dynamical simulations. The analysis of such methods has recently revealed surprising connections between algebra, differential geometry, and computational mathematics. B-series is a central analytical tool of numerical analysis. In the last two decades this tool has found surprising new applications. B-series, discovered in studies of renormalization of quantum field theories by D. Kreimer and Fields medalist A. Connes, showed importance in the development of regularity structures for stochastic differential equations, for which M. Hairer won the Fields medal in 2014.

In recent work, a group around Munthe-Kaas has developed Lie-Butcher series, generalizing Butcher's tools from flat geometries to Lie groups, Klein geometries and homogeneous spaces [20]. This leads to new algebraic structures. An important goal of this WP is to advance these developments, and also develop software for handling the combinatorics of postLie algebras and related structures connecting geometry, algebra and combinatorics. High quality software for symbolic and numerical computations on these structures, combined with modern techniques of automatic differentiation and symbolic algebra, will be invaluable in a wide range of application areas. This includes control theory, geometric robotics, rough paths, information geometry and numerous engineering applications.

Obj6.2: Optimization of computations via algebraic and combinatorial approaches to complexity.

In this objective, we will unify and extend our previous results on efficient algorithms and combinatorial structures [22] to impact the theoretical understanding of computational complexity. Presence of algebraic and combinatorial structures in a computational problem often results in a possibility to reduce the complexity of problems at hand, perhaps even making previously intractable problems tractable. One well-known example of this situation is the fact that the determinant polynomial $\mathrm{Det}(\mathbf{X}_{ij})_{1\le i,j\le n}$ of an $n \times n$ square matrix, whose entries are the indeterminates $\mathbf{X}_{ij}$, is relatively easy to compute (the problem is of the complexity class **VP**), while the permanent polynomial of the same matrix is conjecturally hard to compute (**VNP**). One way to explain this dichotomy is by observing that the determinant polynomial is invariant under the action by conjugation of a large group, namely $\mathbf{GL}_n$ and hence is more `symmetric', compared to the permanent polynomial, which has a smaller symmetry group. The dichotomy between the computational complexities of the determinant and the permanent polynomial is at the heart of the **VP≠VNP** conjecture of Valiant (similar to the famous **P≠NP** conjecture) and the active field of geometric complexity theory [21] that it has spawned. We aim to thoroughly investigate structure of immanents in this context in order to uncover the structural difference between the determinant and the permanent.

For overview purposes, we summarize the relations between the objectives and the research directions:

Directions of Work Packgs	WP1		WP2		WP3		WP4		WP5		WP6	
	Obj11	Obj12	Obj21	Obj22	Obj31	Obj32	Obj41	Obj42	Obj51	Obj52	Obj61	Obj62
RD1: MFC												
RD2: CCM												
RD3: RWA												

2. Impact

2.1: Potential impact of the proposed research

Potential for academic impact of the research project: Within the last two decades, the interaction between pure and computational mathematics developed enormous potential for advancing fundamental, long-standing questions. We aim to take a leading role in expanding this bridge. The MASCOT Center consists at its core of the two research groups within pure mathematics at UiT, which have recently been strengthened through financial support of TFS. The members of these groups have already made significant contributions to the interplay between mathematical structures and computation. This expertise in pure mathematics is complemented by that of computational mathematics, brought to the Center by Munthe-Kaas. The strong theoretical team will be furthermore strengthened by researchers working on computational problems stemming from real world problems. The resulting troika of pure mathematics, computational mathematics and applications will constitute a driving force, and will set new standards: By combining techniques from pure mathematics with computational methods and applications we will arrive at a unique approach, which will open new paths to fundamental questions in mathematics as well as computational problems in applications.

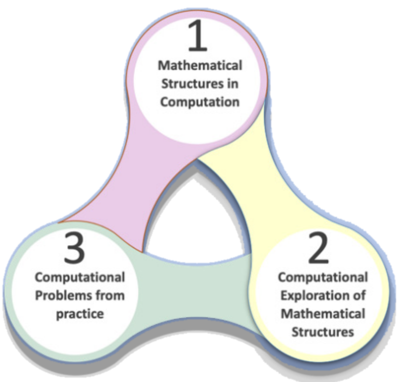

Fig.2: Our Aurora working troika

The timelines for developing the Center are perfect now: Initiated by funding from TFS the research capacities in mathematics at UiT are substantial and will receive an important new component through the Center. Furthermore, the computational capacities available at IMS (IMS possesses several powerful computers in addition to the supercomputer facilities at UiT) allow the above-mentioned cooperation. MASCOT therefore has the potential for carrying out practical computations and deriving theoretical conclusions. This is in line with general developments in mathematics, integrating the effectiveness of symbolic computations in research: while numerical computations operate with rounding and approximations (so that one must justify the results of calculations), symbolic computations keep the rigor standards accepted in fundamental research.

Such approaches opened new horizons recently: These can be highlighted by the following events organized by the members of MASCOT: The Abel Symposia “Mathematics and Computations” 2006 and “Computations and Combinatorics in Dynamics, Stochastics and Control” 2016 co-organized by Munthe-Kaas, “Geometry, Symmetry and Integrability” 2008 co-organized by Kruglikov and “Geometry, Lie theory and Applications” 2019 co-organized by Kruglikov and The, the workshop “Symmetry, Randomness, and Computations in Real Algebraic Geometry” co-organized by Riener at the ICERM, as well as the “MEGA 21” conference at UiT co-organized by Riener, and also the recent TFS symposium “Fundamental interactions between algebra and computation” 2021, where the topics from this proposal were discussed.

Potential for societal impact of the research project: The close cooperation between mathematics, computations and applications within the troika will allow tangible impact beyond fundamental research. Furthermore, in addition to customary dissemination, discussed below, we plan to arrange popularization platforms in close connection with IMS, the Aurora Center DYNAMO, the ERC grant POEMA and the EEA grant GRIEG. A planned symposium at UiT with the Abel Prize winners Lóvász and Wigderson, will help to increase the visibility of the Center and mathematics in general, both at national and international levels. We expect that the activities within the Center, will stimulate interest about mathematics more generally through our dissemination activities, and furthermore will present UiT as an ideal venue for both study and research.

Training and Mentoring: The Center will contribute to PhD education in pure and computational mathematics also on a national level as it will hold biennially regular PhD courses in pure and computational mathematics suitable for students from all Norwegian universities. One of the PhD students will be supervised jointly between Mohammadi and Riener in a cotutelle scheme. Furthermore, at least one additional PhD student financed from the recently awarded Odysseus grant of Mohammadi will spend 50% of the PhD studies at UiT also jointly supervised in a cotutelle. We will create a mentorship program to prepare early career researchers for a career as scientific leaders and, in particular, establish career plans for the early career members. All Center members will engage in the training of master and PhD students, give master/PhD courses and mentor postdocs. Regular common activities will stimulate cooperation and interactions for all early career researchers involved in the project at IMS, IFI and IIT.

Strategic goals: The researchers behind this proposal have made significant scientific contributions in their fields and within MASCOT we will be able to unite a considerable expertise in structure preserving computations based on symmetry methods and combinatorial algorithms. These will have direct relations to cryptography, machine learning, mathematical physics, and engineering. The MASCOT Center will:

1. establish UiT as an internationally recognized node for structural methods,
2. stimulate and strengthen cooperation between fundamental and applied sciences,
3. make these branches pivotal for influence on mathematical science in Norway.

Furthermore, the Center, spanning over four different departments and two different faculties, will establish new important and fruitful connections within UiT. The unique fusion of pure mathematics and computation in combination with applications will create an excellent point of departure for further large RCN/NordForsk applications from the Center, as well as two separate ERC projects. In this way, the Center will serve as the nucleus for a leading national center of excellency that will be established in collaboration with national partners and extensive contacts worldwide.

2.2. Measures for communication and exploitation

Communication with the scientific field: We aim to publish our results in high-rank international peer-reviewed journals. The results will be at once available on ArXiv and other open platforms assuring free access. Furthermore, we will submit the research to open access journals focusing on computational algebra and geometry. We will also publish in journals specializing in computer science and applications. Videos of local lectures at the Center will be distributed via MediaSite UiT, Microsoft Stream and YouTube channels. Additionally, the results of the projects will be regularly presented at international conferences. Here, we will pay attention to provide possibilities for the PhD students and postdocs to participate and present at high-level scientific gatherings. We also plan to arrange a large international conference at UiT during the second half of the project (summer 2024). This will highlight the achievements of the Center and strengthen the prospects for propagation of its results. Finally, we aim to implement the computational techniques that arise during the project in free open domain software packages.

Communication with the public: Through the Aurora Center we will actively engage in a promotion of mathematics. This will be achieved by hosting a regular series of general audience talks devoted to highlight applications of mathematics and computations to other fields of science ("Math Matters"). Further, we will inform the public about the results of our studies and the impact of algorithmic mathematics to everyday life. We aim to be actively involved in creating a wider interest in mathematics. This will be achieved by giving public lectures (UiT's Lørdagsuniversitetet, Academia Borealis Nord-Norges Vitenskapsakademi, special arrangements at Tromsø Science Center during Forskingsdagene), and by contacting local schools. In addition, we will continue our collaboration with UiT's Skolelab and regularly invite highschool students to actively explore mathematics. Furthermore, we hope to be able to use the project communication also to reduce the gender imbalance in mathematics by actively promoting a gender inclusive image of mathematics through thematic events[7]. Finally, we will have an updated Center homepage with mirrors in social media. On this homepage, we will document the ongoing project work, post additional explanatory videos and regularly publish the results of our studies.

DORA and open science: The project team supports the DORA declaration and will make all research data, computer codes and publications in the Center open access. We will employ using the ArXiv, Cristin and Munin databases, as well as UiT Open Research Data and popularization sources.

3. Implementation

3.1. Project manager and project group

The Center will be governed by a managing team consisting of a director, together with work package leaders. The director has responsibility for all Center activities, in particular, the fulfillment of its scientific ambitions. In the first two-year period, the director of the Center will be Boris Kruglikov. Cordian Riener will act as a vice-director during the initial two years and continue as a director for the second two-year term.

Name	*WP1*	*WP2*	*WP3*	*WP4*	*WP5*	*WP6*	*Nationality*	*Age*	*Publications*	*Citations*	*h-index*
Chiara Bordin (CB)			✓		✓		Italian	37	19	259	5
Viveka Erlandsson (VE)				✓			Swedish	40	13	89	5
Trygve Johnsen (TJ)	✓						Norwegian	64	37	351	11
Boris Kruglikov (BK)		✓		✓		✓	Russian	50	107	1351	20
Fatemeh Mohammadi (FM)	✓		✓			✓	Iranian	38	46	570	15
Hans Munthe-Kaas (HMK)	✓	✓				✓	Norwegian	59	52	2960	23
Andrey Piatnitski (AP)				✓	✓		Russian	65	135	2698	28
Cordian Riener (CR)	✓		✓			✓	Austrian	40	20	221	8
Wei Solvang (WS)					✓		Norwegian	52	116	730	15
Dennis The (DT)		✓			✓		Canadian	41	22	293	8

Project group responsibility, diversity and competence (data based on Google Scholar)

Boris Kruglikov is a leader of the Pure Mathematics group at IMS, as well as the leader of the TFS-project Pure Math in Norway and the GRIEG project SCREAM on symmetry methods. Cordian Riener is the PM of a TFS project SymRAG on symmetry in algorithmic questions, PI in the MSC-INT project POEMA on algebraic structures in optimization and a member of the board of the Norwegian Math Society. Trygve Johnsen is a former head of IMS and a leader of a NFR and ISP projects, in part on data security. Hans Munthe-Kaas is the leader of the Norwegian Math Society, chair of the Abel Committee, member of the Academy of Science and Letters, and an expert in numerical computations. Chiara Bordin is an expert in mathematical optimization in computer science, applied to energy and power systems, and a member of ARC. Wei Solvang is a former head of IIT Department at UiT-Narvik and PI in several grants (H2020, EuroStar, etc) on Advanced Robotics and Digital Technologies. Viveka Erlandsson is an expert in geometry and a member of Equality and Diversity committee in Bristol. Dennis The and Andrey Piatnitski are experts in differential geometry and analysis, and PI in several research projects. Fatemeh Mohammadi is an expert in algebraic optimization, the PI in an Odysseus Group project awarded by the Belgian Research Foundation in Flanders, and a fellow of EPSRC UK.

Professorship: The Department of Mathematics and Statistics will support the Center by covering 2 professor positions. Firstly, Munthe-Kaas will be hired in a permanent position and join UiT. Secondly, the position of Riener, covered by TFS for the first 2 years of the project duration, will be converted into a permanent one.

International Adjunct Professors: The Center will extend by 2 years (2024-2025) the Prof-II positions of Fatemeh Mohammadi and Viveka Erlandsson, who are currently hired by IMS within TFS project SymRAG. One more Prof-II position will be created to support the activity of the team.

National and international collaborations: The team will involve scientific exchange on a national and international level. The following are the main collaborators envisioned within the different WPs:

- WP1: C Hollanti (Finland), S Ghorpade (India), H Verdure (UiT-IMS);
- WP2: B Doubrov (Belarus), I Markina (UiB), K Ebrahimi-Fard (NTNU);
- WP3: G Blekherman (USA), B Mourrain (France), S Mishra (Estonia);
- WP4: E Ferapontov (UK), R Vitolo (Italy), J Souto (France);
- WP5: Y Costa-Salas (Spain), E Zhizhina (Russia), G Chechkin (Russia);

[7] For example: https://womeninmath.net/venue/tromso-norway/

- WP6: S Basu (USA), I Miklós (Hungary), G Fløystad (UiB);

These partners will be involved in both our research as well as our outreach activities.

Gender balance: 45% of the key researchers in the project (PMs & PIs) are women, which is a sharp contrast to the traditional gender imbalance in the natural sciences. Further improvement of the gender balance of mathematics at UiT is one of the missions of MASCOT. The female research leaders involved in the Center will act as role models both for early-career female mathematicians in Norway, and potentially also for young girls in the audience of our outreach activities. We will help to reduce specific career hurdles for gender minorities and keep working towards gender equality in all activities related to our Center.

3.2 Project organisation and management

The MASCOT Aurora Center will be hosted by the Department of Mathematics and Statistics (IMS) and managed by the Faculty for Science and Technology. The management team, comprised of the director and one principal investigator per work package, oversees all research, training, outreach, and dissemination. They have the responsibility to allocate resources among the research themes, identify new projects, and promote synergies. Each work package is headed by a leader and a co-leader, who are responsible for research, proposals, as well as for hiring and overseeing the progress of PhD students and postdocs within the WP.
A scientific advisory committee consisting of Ragni Piene (Norway), Peter Olver (USA) and Peter Bürgisser (Switzerland) will meet with the Center annually, alternately virtually on Zoom and physically in Tromsø, to give independent feedback on research, management, and funding. We plan regular focused working groups, joint seminars (virtual and physical) as well as one workshop per year for all project participants. One large conference will unite our team and partners with world experts in the field and the advisory committee members to discuss the progress and development of the Center.

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